Topic: Rolle's theorem

Question: Which of these is not part of Rolle's Theorem?

Answer choices:

A The function $f(x)$ must be continuous on the closed range $[a, b]$.

B That the function $f(x)$ meets the condition $f(a) = f(b)$ for the given range $[a, b]$.

C The function $f(x)$ can be integrated over the open interval (a, b).

D The function $f(x)$ is differentiable on the open interval (a, b).

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Solution: C

Rolle's Theorem requires three conditions be met in order for its conclusion to be true.

- The function $f(x)$ must be continuous on the closed interval $[a, b]$
- The function $f(x)$ is differentiable on the open interval (a, b)
- The function $f(x)$ meets the condition $f(a) = f(b)$ for the interval $[a, b]$

If these three conditions are met, Rolle's Theorem states that there must exist a point c within the interval (a, b) where $f'(c) = 0$.

Answer choice C is the correct choice because integration plays no part in the conditions or conclusions of Rolle's Theorem.

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Topic: Rolle's theorem

Question: Does the function meet the criteria of Rolle's Theorem?

$$f(x) = x^2 - x + 6$$

on the interval $[0,1]$

Answer choices:

A Yes, it's continuous and differentiable over the interval and $f(0) = f(1)$.

B Yes, it's continuous and $f(0) \neq f(1)$.

C No, it's not differentiable over the interval.

D No, it's discontinuous and $f(0) \neq f(1)$.

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Solution: A

Rolle's Theorem requires three conditions be met in order for its conclusion to be true.

- The function $f(x)$ must be continuous on the closed interval $[a, b]$
- The function $f(x)$ is differentiable on the open interval (a, b)
- The function $f(x)$ meets the condition $f(a) = f(b)$ for the interval $[a, b]$

If these three conditions are met, Rolle's Theorem states that there must exist a point c within the interval (a, b) where $f'(c) = 0$.

Let's check the first condition of Rolle's theorem by testing the continuity of the function over the given interval.

Since this is a polynomial function, and we know that polynomial functions are continuous for all real numbers, we know that the function is continuous on the interval $[0,1]$.

Let's check the second condition of Rolle's theorem by testing the differentiability of the function over the given interval. Taking the derivative,

$$f'(x) = 2x - 1$$

we can see that the function is differentiable.

To check the third condition of Rolle's theorem, we need to confirm that $f(0) = f(1)$.

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$$f(0) = (0)^2 - (0) + 6$$

$$f(0) = 6$$

and

$$f(1) = (1)^2 - (1) + 6$$

$$f(1) = 6$$

Since $f(0) = 6 = f(1)$, we've confirmed that this function over the given interval meets all three conditions of Rolle's theorem.

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Topic: Rolle's theorem

Question: Use Rolle's theorem to find the coordinate point in the interval where the function has a horizontal tangent line.

$$f(x) = -x^2 + 4x + 16$$

on the interval $[0,4]$

Answer choices:

A $(0,4)$

B $(-2,20)$

C $(0,-4)$

D $(2,20)$

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Solution: D

Rolle's Theorem requires three conditions be met in order for its conclusion to be true.

- The function $f(x)$ must be continuous on the closed interval $[a, b]$
- The function $f(x)$ is differentiable on the open interval (a, b)
- The function $f(x)$ meets the condition $f(a) = f(b)$ for the interval $[a, b]$

If these three conditions are met, Rolle's Theorem states that there must exist a point c within the interval (a, b) where $f'(c) = 0$. First, we need to meet the criteria for Rolle's Theorem and then we can solve for point c.

Let's check the first condition of Rolle's theorem by testing the continuity of the function over the given interval.

Since this is a polynomial function, and we know that polynomial functions are continuous for all real numbers, we know that the function is continuous on the interval $[0,4]$.

Let's check the second condition of Rolle's theorem by testing the differentiability of the function over the given interval. Taking the derivative,

$$f'(x) = -2x + 4$$

we can see that the function is differentiable.

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To check the third condition of Rolle's theorem, we need to confirm that $f(0) = f(4)$.

$$f(0) = -(0)^2 + 4(0) + 16$$

$$f(0) = 16$$

and

$$f(4) = -(4)^2 + 4(4) + 16$$

$$f(4) = 16$$

Since $f(0) = 16 = f(4)$, we've confirmed that this function over the given interval meets all three conditions of Rolle's theorem.

Now we can find the point c by solving the equation $f'(c) = 0$.

$$-2x + 4 = 0$$

$$2x = 4$$

$$x = 2$$

Therefore, $c = 2$.

To find the coordinate point associated with $c = 2$, we'll plug it back into our original function and get

$$f(2) = -(2)^2 + 4(2) + 16$$

$$f(2) = 20$$

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The conclusion of Rolle's theorem tells us that the function has a horizontal tangent line at the point c inside the interval $[a, b]$, so we can say that the function has a horizontal tangent line at the point $(2{,}20)$.

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